Analysis of the Robustness of Degree Centrality against Random Errors in Graphs

Sho Tsugawa[1] and Hiroyuki Ohsaki[2]

[1] University of Tsukuba, Tsukuba, Ibaraki 305-8573, Japan
s-tugawa@cs.tsukuba.ac.jp
[2] Kwansei Gakuin University, Sanda, Hyogo 669-1337, Japan
ohsaki@kwansei.ac.jp

Abstract. Research on network analysis, which is used to analyze large-scale and complex networks such as social networks, protein networks, and brain function networks, has been actively pursued. Typically, the networks used for network analyses will contain multiple errors because it is not easy to accurately and completely identify the nodes to be analyzed and the appropriate relationships among them. In this paper, we analyze the robustness of centrality measure, which is widely used in network analyses, against missing nodes, missing links, and false links. We focus on the stability of node rankings based on degree centrality, and derive Top_m and $\mathrm{Overlap}_m$, which evaluate the robustness of node rankings. Through extensive simulations, we show the validity of our analysis, and suggest that our model can be used to analyze the robustness of not only degree centrality but also other types of centrality measures. Moreover, by using our analytical models, we examine the robustness of degree centrality against random errors in graphs.

1 Introduction

Research on network analysis, which is used to analyze large-scale and complex networks such as social networks, protein networks, and brain function networks, has been actively pursued [1,6,8,20–22]. In network analysis, relationships among entities in the real world are represented by a graph. In social network analysis (SNA), individuals are represented as nodes in a graph, and the social ties among them, such as similarities, social relations, interactions, and flows, are represented as links [6, 22]. In brain function network analysis, brain regions are represented as nodes, and temporal correlations in activity among them are represented as links [20].

Among various indices proposed for network analysis, centrality measures (e.g., degree centrality, betweenness centrality, closeness centrality, and eigenvector centrality) [4, 11] have been widely used in actual analyses [3, 5, 25]. Centrality measures are indices that express the influence of one node on others, and such measures have been used for various purposes, such as discovering which person plays a central role in a community [3, 5] and inferring which brain regions are important for the task of interest [25].

Typically, the graphs used for network analyses will contain multiple errors because it is not easy to accurately and completely identify the entities to be analyzed and the appropriate relationships among them [7, 9, 14–16, 19]. For instance, graphs used in SNA can contain several errors of different types, such as *missing nodes*, *missing links*, and *false links*. In traditional SNA, graphs are generated from the results of questionnaires, and so non-responses and inaccurate answers will cause such errors [24]. Even in recent SNA used for analyzing online social networks, such errors can be present due to sampling bias and restrictions on social network data, which is typically accessed by means of application programming interfaces. In biological network analyses, such as analyses of protein interaction networks and gene regulatory networks, graphs often contain errors such as missing links and false links as a result of measurement errors [19, 23].

Several analyses on the robustness of centrality measures used for network analyses against errors in the graphs (simulated as noise created by random addition and deletion of nodes and links) have been performed [7, 9, 12–17, 19]. In [7, 16], how centrality measures of nodes in networks are affected by the random addition and deletion of nodes and links is experimentally investigated. Robustness of centrality measures against link weight noises has also been experimentally investigated, such as in [13, 17].

Most existing studies use an experimental approach to understand the robustness of centrality measures, but some recent studies adopt a theoretical approach. Ghoshal *et al.* [12] analyze node-ranking stability based on the PageRank algorithm against random rewiring of links. Platig *et al.* [19] develop an analytical model to quantify the robustness of degree centrality against link errors (i.e., missing links and false links). They derive correlation coefficients r between the degree measures of the ground-truth graph and those of graphs with errors.

Our study builds on prior work and contributes to developing an analytical model that can be used to quantify the robustness of centrality measures. Since one of the most typical errors in network analysis is missing nodes [7, 9, 24], we extend the model of [19] to include these, and analyze the robustness of degree centrality against missing nodes as well as against missing links and false links. As discussed in the previous works [7, 19], centrality measures are used mainly for node ranking. We therefore focus on the stability of node ranking and derive Top_m and $\mathrm{Overlap}_m$, which evaluate the robustness of node rankings [7, 16, 17, 19]. Through extensive simulations, we show the validity of our analysis. Moreover, by using our analytical models, we examine the robustness of centrality measures against random errors in graphs.

The remainder of this paper is organized as follows. Section 2 introduces related work. In Section 3, we analyze the robustness of degree centrality against three types of errors (i.e., missing nodes, missing links, and false links). Section 4 examines the validity of our analysis through comparison between numerical examples of our analysis and results of simulations, and also discusses the robustness of centrality measures against random errors in graphs. Finally, Section 5 contains our conclusions and a discussion of future work.

2 Related Work

Most existing studies use a simulation to understand the robustness of centrality measures by adding errors to a ground-truth graph and investigating the relation between the centrality measures of the ground-truth graph and those of the graphs with errors [7, 9, 14–16]. In contrast, some recent studies use a theoretical approach [12, 19].

Ghoshal *et al.* [12] analytically derive the conditions under which node ranking according to PageRank is stable against random rewiring of links. Platig *et al.* [19] investigate the robustness of centrality measures against link errors (i.e., missing links and false links) through simulations and theoretical analysis. In their analysis, the correlation coefficients r of degree centrality between a ground-truth graph and graphs with errors are derived.

The only type of error studied in Ghoshal *et al.* [12] is link rewiring, and typical errors such as node and link addition and deletion are not considered. Platig *et al.* [19] investigate the robustness of centrality measures against link errors typical in network analysis, but the effect of node deletion, which is also a typical error in network analysis [7, 9, 24], is not studied. Moreover, the stability of node ranking based on centrality measures is investigated in their simulations, but a theoretical analysis of the node ranking stability is not performed. The correlation coefficients r of centrality measures, which are theoretically analyzed in [19], and Top_m and $\mathrm{Overlap}_m$, which are studied in this paper, exhibit different tendencies [19]. In this paper, we extend the model of [19] and use this extended model to analyze the robustness of centrality measures, as measured by node ranking stability based on degree centrality, against missing nodes, missing links, and false links.

3 Analysis

We analyze the consistency of node ranking based on degree centrality, comparing an undirected unweighted graph $G = (V, E)$ with a graph G_e that is a copy of G with random errors introduced. We analyze the robustness of degree centrality against three types of errors, which correspond to the following operations: link deletion, node deletion, and link addition. The link deletion error independently deletes each link of graph G with probability α; the node deletion error independently deletes each node in graph G and all links associated to that node with probability β; and the link addition error randomly adds $\gamma|E|$ links to graph G, where $|E|$ is the number of links in graph G. We assume that the graph G has an arbitrary degree distribution [18], and that the degree of each node in graph G $(k_1, k_2, \ldots, k_{|V|})$ is known, where $|V|$ is the number of nodes in graph G.

We rank all the nodes in graphs G and separately in G_e by sorting the nodes in descending order of their degree centrality, and we analyze the node ranking consistency between graphs G and G_e. We particularly focus on the ranking of highly ranked nodes, and derive expected values of Top_m and $\mathrm{Overlap}_m$, which are used to evaluate the robustness of node ranking. Top_m is the probability that the most central node in graph G is ranked in the top m most central nodes in graph G_e [7, 16, 17]. $\mathrm{Overlap}_m$ is the overlap between the top m most central nodes in graph G and those in graph G_e. More specifically, let $U_m(G)$ be the set of the m most central nodes in graph G; then, $\mathrm{Overlap}_m$ is defined as $|U_m(G) \cap U_m(G_e)|/m$ [7, 16, 17, 19]. These measures are used in the simulation studies [7, 16, 17, 19] to evaluate the robustness of centrality measures. Table 1 shows the definitions of symbols used in this paper.

Table 1. Definitions of symbols used in this paper

G	Unweighted undirected graph
G_e	Unweighted undirected graph with errors
V	Set of nodes in graph G
E	Set of links in graph G
k_i	Degree of a node whose degree is the ith largest in graph G
v_i	Node whose degree is the ith largest in graph G
V_i	Subset of V defined as $V - \{v_i\}$
α	Probability of deleting each link in graph G
β	Probability of deleting each node in graph G
γ	Ratio of links added to graph G
$p(l\|k)$	Probability that a node with degree k in graph G has degree l in graph G_e
$P(l\|k)$	Probability that a node with degree k in graph G has degree l or less in graph G_e
$\overline{P}(l\|k)$	Probability that a node with degree k in graph G has degree more than l in graph G_e
$t_{i,j}$	Probability that node v_i has the jth largest degree in graph G_e
Top_m	Probability that node v_1 is ranked in the top m most central nodes in graph G_e
$\mathrm{Overlap}_m$	Overlap between the top m most central nodes in graph G and those in graph G_e

Let $p(l|k)$ be the probability that a node with degree k in graph G has degree l in graph G_e. We derive Top_m and $\mathrm{Overlap}_m$ by using $p(l|k)$. In what follows, v_i denotes a node whose degree is the ith largest in graph G, and k_i denotes the degree of node v_i.

To obtain Top_m and $\mathrm{Overlap}_m$, we first obtain the probability that node v_i has the jth largest degree in graph G_e, which is denoted as $t_{i,j}$. First, let us consider $t_{i,1}$, which is the probability that node v_i has the largest degree in graph G_e. Node v_i has the largest degree in graph G_e if and only if the degree of each node is less than or equal to k_i, and therefore $t_{i,1}$ is given by

$$t_{i,1} = \sum_{l=0}^{|V|-1} p(l|k_i) \prod_{r \neq i} P(l|k_r), \tag{1}$$

where $P(l|k)$ is the probability that a node with degree k in graph G has degree l or less in graph G_e; this is given by the following equation.

$$P(l|k) = \sum_{s=0}^{l} p(s|k) \tag{2}$$

Next, let us consider the case with $j > 1$. Node v_i has the jth largest degree in graph G_e if and only if $(j-1)$ nodes have a higher degree than node v_i in graph G_e and all other nodes have a weakly lower degree than node v_i. Here, we define the following symbols.

$$\overline{P}(l|k) = 1 - P(l|k) \tag{3}$$

$$Q(l,k_i,S) = \begin{cases} \overline{P}(l|k_i) & v_i \in S \subset V \\ P(l|k_i) & \text{otherwise} \end{cases} \tag{4}$$

$$V_i = V - \{v_i\} \tag{5}$$

Then, $t_{i,j}$ is given by

$$t_{i,j} = \sum_{l=0}^{|V|-1} p(l|k_i) \sum_{X \in \binom{V_i}{j-1}} \prod_{r \neq i} Q(l, k_r, X), \tag{6}$$

where $\binom{V_i}{K}$ is the set of all subsets of V_i which have a given size K.

Top_m is the sum of the probability that node v_1 has the largest degree in graph G_e, the probability that node v_1 has the second largest degree in graph G_e, ..., and the probability that node v_1 has the mth largest degree in graph G_e. Symbolically,

$$\text{Top}_m = \sum_{j=1}^{m} t_{1,j}. \tag{7}$$

Additionally, we define $T_{i,j}$ as follows.

$$T_{i,j} = \sum_{s=1}^{j} t_{i,s} \tag{8}$$

Since the expected number of overlapping nodes between the top m most central nodes in graph G and those in graph G_e is $\sum_{i=1}^{m} T_{i,m}$, Overlap_m is then given by

$$\text{Overlap}_m = \frac{\sum_{i=1}^{m} T_{i,m}}{m}. \tag{9}$$

We next derive $p(l|k)$, the probability that a node with degree k in graph G has degree l in graph G_e.

First, let us consider the case with *link deletion*. A node with degree k in graph G has degree l in graph G_e if and only if $(k-l)$ links are deleted from the node. The probability distribution of the number of deleted links follows the binomial distribution, and therefore, as also shown in [19], the probability that a node with degree k in graph G has degree l in graph G_e is given by

$$p_D(l|k) = \binom{k}{k-l} (1-\alpha)^l \alpha^{k-l}. \tag{10}$$

Next, let us consider the case with *node deletion*. In this case, similarly to the case with *link deletion*, the probability that s links are deleted from a node with degree k follows the binomial distribution. Hence, the probability that a node with degree k in graph G has degree l in graph G_e is given by

$$p_v(l|k) = \begin{cases} (1-\beta)\binom{k}{k-l}(1-\beta)^l \beta^{k-l} & l > 0 \\ \beta + (1-\beta)\beta^k & l = 0. \end{cases} \tag{11}$$

Next, let us consider the case with *link addition*. The probability that s links are added to a node with degree k is approximated by the Poisson distribution when the number of nodes $|V|$ is sufficiently large. Hence, as shown in [19], the probability that a node with degree k in graph G has degree l in graph G_e is approximated by

$$p_a(l|k) \simeq \frac{u^{l-k}}{(l-k)!} e^{-u}, \tag{12}$$

where u is the average number of added links per node, and is defined as $u = 2|E|\gamma/|V|$.

Next, let us consider the case with both *link deletion* and *link addition*. The probability that a node with degree k in graph G has degree l in graph G_e is derived in [19], and given by

$$p_{da}(l|k) = \sum_r \frac{u^r e^{-u}}{r!} \binom{k}{k+r-l} (1-\alpha)^{l-r} \alpha^{k+r-l}. \tag{13}$$

Finally, we consider the case with all of *link deletion*, *link addition*, and *node deletion*. A node with degree k in graph G has degree l in graph G_e if and only if r links are added, s adjacent nodes are deleted, and $(k+r-s-l)$ links are deleted from the node. Hence, combining Eqs. (11) and (13), the probability that a node with degree k in graph G has degree l in graph G_e is given by the following equation.

$$p(l|k) = \begin{cases} (1-\beta)\sum_r \frac{u^r e^{-u}}{r!} \sum_s \binom{k}{s} (1-\beta)^{k-s} \beta^s \\ \times \binom{k-s}{k+r-s-l} (1-\alpha)^{l-r} \alpha^{k+r-s-l} & l > 0 \\ \beta + \\ (1-\beta) e^{-u} \sum_s \binom{k}{s} (1-\beta)^{k-s} \beta^s \alpha^{k-s} & l = 0 \end{cases} \tag{14}$$

Note that we can also obtain a correlation coefficient r between degrees in graph G and those in graph G_e by using Eq. (14) and the model in [19].

4 Numerical Examples and Simulation Results

In this section, we examine the validity of our analysis by comparison between numerical examples of our analysis and the results of simulations. Moreover, we also discuss the effects of missing nodes, missing links, and false links on node rankings that are based on degree centrality.

As the ground-truth graph G, we use random graphs generated with the ER (Erdös–Rényi) model [10] and scale-free graphs generated with the BA (Barabási–Albert) model [2]. The number of nodes is 200, and the average degree of a node is 5 in the ER model and 2 in the BA model. In our simulations, we obtain graph G_e by deleting each link with probability α, deleting each node with probability β, and adding $\gamma|E|$ links between randomly selected pairs of unlinked nodes. For each graph G, we obtain 200 graphs for G_e, and calculate Top_m and Overlap_m. We generate 100 different initial graphs G, and obtain averages of Top_m and Overlap_m. We also obtain Top_m and Overlap_m by using our analytical models from degrees of nodes in graph G and the parameters α, β, and γ. In what follows, lines in the figures represent the results of analysis, and dots represent results of simulation.

We first investigate Top_m and Overlap_m when only a single type of error is contained in the graphs. Namely, we obtain Top_m and Overlap_m while two of α, β, and γ are fixed at 0 and the other parameter is changed. Figures 1, 2, and 3 show the results when changing α, β, and γ, respectively; Top_1, Top_3, and Overlap_3 are used to characterize the results. From these results, we can confirm that the results of analysis are in good agreement with the simulation results. These results show the validity of our analysis.

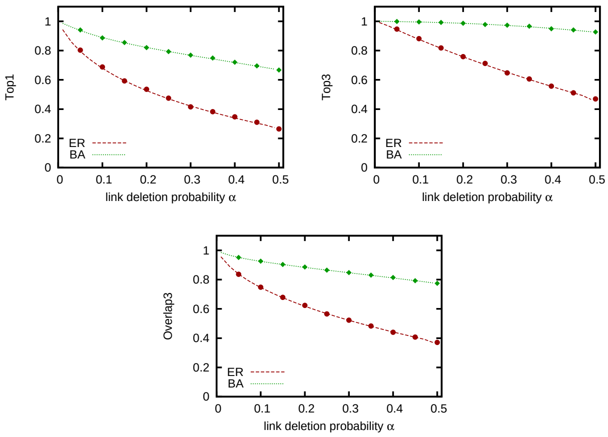


Fig. 1. Link deletion probability α vs. Top_1, Top_3, and Overlap_3

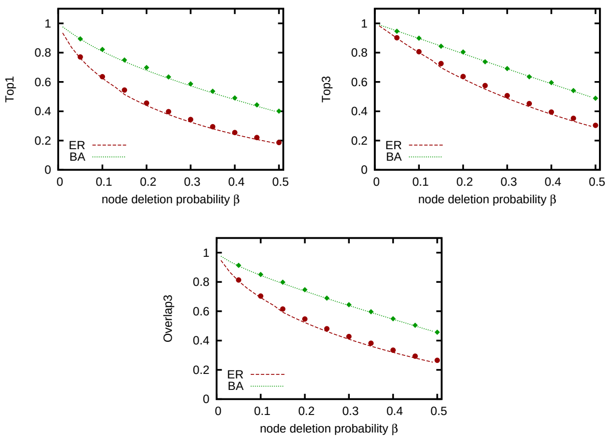


Fig. 2. Node deletion probability β vs. Top_1, Top_3, and Overlap_3

We next investigate Top_m and Overlap_m when multiple types of errors are contained in graphs. We focus on two cases: a case with both missing links and false links, and a case with missing links and missing nodes. A typical example of the first case is the case of constructing protein interaction networks, where measurement error causes both missing links and false links. A typical example of the latter case is the case of constructing a social network, where incomplete data causes both missing links and missing nodes. Figure 4 shows Top_1, where errors of both missing links and false links are contained in graphs, and Fig. 5 shows Top_1, where errors of both missing links and

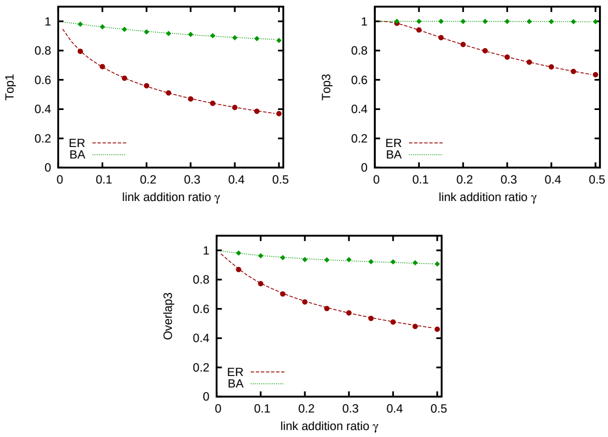


Fig. 3. Link addition ratio γ vs. Top_1, Top_3, and $\mathrm{Overlap}_3$

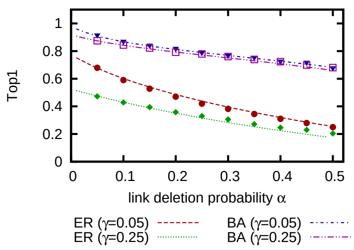


Fig. 4. Top_1 when errors of both missing links and false links are contained in graphs

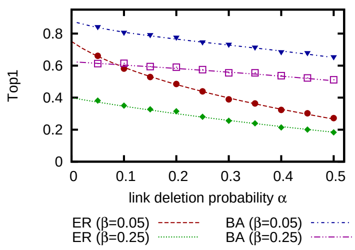
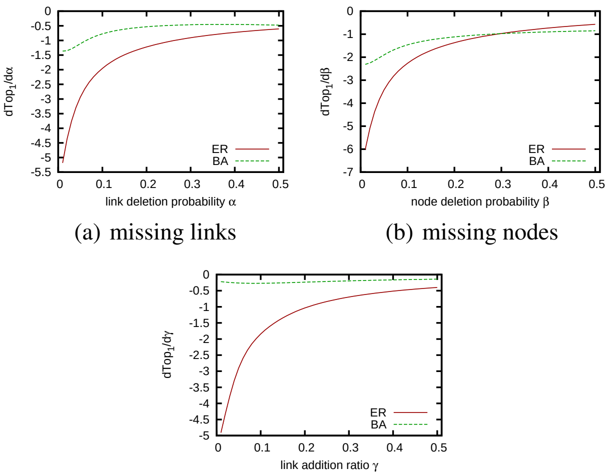


Fig. 5. Top_1 when errors of both missing links and missing nodes are contained in graphs

missing nodes are contained in graphs. These figures show that analytical results and simulation results coincide closely. These results show the validity of our analysis when multiple types of errors are contained in graphs.

From these results, as previously shown in the simulation studies [7, 16, 19], we can observe non-negligible effects of random errors in graphs on node rankings based on degree centrality. Graphs generated according to the BA model are more robust than those generated according to the ER model, but in the particular case with missing nodes, Top_1, Top_3, and $\mathrm{Overlap}_3$ decrease almost linearly. Thus, our analysis gives theoretical confirmation of the results from previous works.

We further analyze the robustness of degree centrality by using analytical models. We differentiate Top_1 with respect to the error rates α, β, and γ, and investigate the effects of each type of error on the node ranking. Figure 6 shows the derivation of Top_1 with respect to the error rates α, β, and γ. These figures show the relation between an increase in the error rate and a decrease in the accuracy of detecting most central



(a) missing links (b) missing nodes

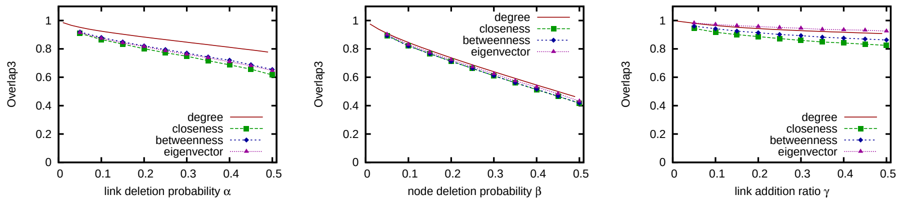
(c) false links

Fig. 6. Derivation of Top_1 with respect to α, β, and γ: in the panels, one parameter is changed and the other two parameters are fixed to 0.

node according to degree centrality. From these figures, we can find that, for instance, in graphs generated from the BA model, a 1% increase in the rate of missing nodes causes an approximately 2% decrease in Top_1 when the node deletion probability is less than 0.1. Our models reveal the relation between the increase of the error ratio and the decrease in accuracy of centrality.

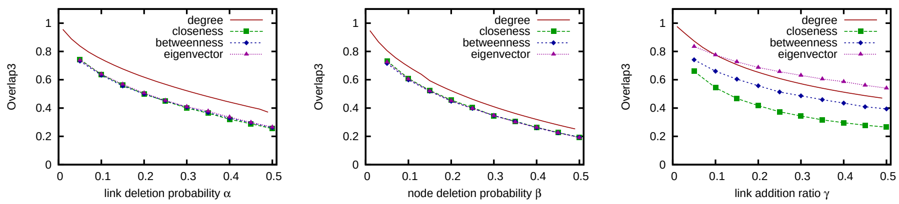
Finally, we investigate the robustness of other types of centrality measures (specifically, betweenness, closeness, and eigenvector centralities) through simulations. Due to space limitation, we show the results of $\mathrm{Overlap}_3$ only. Figures 7 and 8 show $\mathrm{Overlap}_3$ of different types of centrality measures when α, β, and γ are changed in the BA model (Fig. 7) and the ER model (Fig. 8), respectively. For comparison purposes, the analytical results of $\mathrm{Overlap}_3$ of degree centrality are also shown on the graphs.

Figure 7 shows that $\mathrm{Overlap}_3$ of the four types of centrality is similar among graphs generated with the BA model. Figure 8 shows that in the ER model, the magnitudes of $\mathrm{Overlap}_3$ are different, but the curves of $\mathrm{Overlap}_3$ are of similar shape for the four types of centrality measures. We observed (not shown here) that Top_1 and Top_3 also exhibit similar tendencies. These results indicate that the four types of centrality measures have similar robustness, particularly in graphs generated according to the BA model. This suggests that analytical models of degree centrality can be used to predict the robustness of other types of centrality measures. The cause of the similar robustness among the four types of centrality measures can be attributed to the high correlation among the centrality measures.



(a) missing links (b) missing nodes (c) false links

Fig. 7. Overlap_3 of the four types of centrality measures (degree, closeness, betweenness, and eigenvector centralities) in graphs generated according to the BA model: Overlap_3 of degree centrality is obtained by our analysis, and values with the other measures are obtained by simulation



(a) missing links (b) missing nodes (c) false links

Fig. 8. Overlap_3 of the four types of centrality measures (degree, closeness, betweenness, and eigenvector centralities) in graphs generated according to the ER model: Overlap_3 of degree centrality is obtained by our analysis, and values with the other measures are obtained by simulation

5 Conclusion and Future Works

We analyzed the robustness of degree centrality against missing nodes, missing links, and false links. We extended the model of [19], and derived Top_m and Overlap_m, which were used to evaluate the robustness of node rankings, and showed the validity of the analysis. Moreover, through extensive simulations, we showed that the four types of popular centrality measures (degree, closeness, betweenness, and eigenvector centralities) exhibit similar robustness, which suggests that our model can be used to analyze the robustness of not only degree centrality but also other types of centrality measures.

As future work, we plan to analyze the robustness of centrality measures other than degree centrality. Investigating the effects of other types of errors is also important future work. This paper focuses on uniform errors, but in actual network analyses, non-uniform errors arise. As an example, biased sampling is a known cause of non-uniform errors, and such types of errors are of interest to network researchers.

Acknowledgments

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